Elementary Algebra
Yellow Cover
50 questions, 45 minutes

Skills Assessed: Arithmetic operations, polynomials, linear equations and inequalities, quadratic equations, graphing, rational expressions, exponents and square roots, geometric measurement, word problems

Recommended Placement

MATH 100: Elementary Algebra
MATH 110: Elementary Geometry
MATH 120: Intermediate Algebra

There are 4 levels of math assessment. If you test at a level too high for your current skills, you will not receive a placement. It will be recommended that you retest at a lower level. Choose a test that you are comfortable with when selecting which math test to take.

Take your assessment results with you when you meet with a counselor.

Your Results Report is an important document. Save it!
Topic 1: Arithmetic Operations

A. Fractions

Simplifying fractions:

Example: Reduce 27/36.
\[ \frac{27}{36} = \frac{9 \times 3}{9 \times 4} = \frac{3}{4} \]
(Note that you must be able to find a common factor—In this case 9—in both the top and bottom in order to reduce.)

1 to 3: Reduce:
1. \[ \frac{13}{39} = \frac{3}{13} \]
2. \[ \frac{26}{65} = \frac{2}{5} \]

Equivalent fractions:

Example: 3/4 is equivalent to how many eighths?
\[ \frac{3}{4} = \frac{3}{4} \]
\[ \frac{1 \times 3}{4 \times 3} = \frac{3}{12} \frac{2 \times 3}{4 \times 3} = \frac{6}{12} \frac{2 \times 3}{4 \times 3} = \frac{3}{8} \]

4 to 5: Complete:
4. \[ \frac{3}{4} = \frac{72}{96} \]
5. \[ \frac{1}{5} = \frac{20}{100} \]

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

Example: 5/6 and 8/15
First find LCM of 6 and 15:
6 = 2 \cdot 3
15 = 3 \cdot 5
LCM = 2 \cdot 3 \cdot 5 = 30, so
6 = 2 \cdot 3 \cdot 5 \cdot 2 = 30
8 = 2 \cdot 3 \cdot 5 \cdot 2 = 30

6 to 7: Find equivalent fractions with the LCD:
6. \[ \frac{5}{6} \text{ and } \frac{3}{5} \]
7. \[ \frac{1}{6} \text{ and } \frac{7}{12} \]

8. Which is larger, 5/7 or 3/4?
(Hint: find LCD fractions)

Adding, subtracting fractions: If denominators are the same, combine the numerators:

Example: \[ \frac{1}{10} - \frac{1}{10} = \frac{7}{10} - \frac{1}{10} = \frac{6}{10} = \frac{3}{5} \]

9 to 11: Find the sum or difference (reduce if possible):
9. \[ \frac{3}{4} + \frac{2}{3} = \frac{11}{12} \]
10. \[ \frac{5}{6} + \frac{1}{6} = \frac{6}{6} = 1 \]

If denominators are different, find equivalent fractions with common denominators, then proceed as before:

Example:
\[ \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = \frac{1}{3} \]
\[ \frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4} = \frac{1}{4} \]

12. \[ \frac{2}{3} - \frac{2}{3} = \frac{1}{3} \]
13. \[ \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \]

Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible:

Example: \[ \frac{2}{3} \cdot \frac{3}{4} = \frac{6}{12} = \frac{1}{2} \]

14. \[ \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2} \]
15. \[ \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \]
16. \[ (\frac{1}{2})^2 = \frac{1}{4} \]

Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom of the big fraction by the LCD of both:

Example:
\[ \frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2} \]
\[ \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3} \]
\[ \frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2} \]
\[ \frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \cdot \frac{3}{1} = \frac{3}{2} \]

B. Decimals

Meaning of places: In 324,519, each digit position has a value ten times the place to its right. The place to the left of the point is the whole number part. Right of the point, the places have values: tenths, hundredths, etc., so 324,519 = (3 \times 100) + (2 \times 10) + (4 \times 1) + (5 \times 1/10) + (1 \times 1/100) + (9 \times 1/1000).

23. Which is larger, .59 or .7?

To add or subtract decimals, like places must be combined (line up the points):

Example:
\[ 1.23 - 1.13 = 0.10 \]
\[ 4.3 + 1.3 = 5.6 \]
\[ 6.04 - 2.14 = 3.90 \]

24. \[ .54 + .78 = \]
25. \[ .36 + .63 = \]
26. \[ .4 - .3 + .001 - .01 + .1 = \]
27. \[ .154 - .168 = \]

Multiplying decimals:

Example:
\[ .3 \times .5 = .15 \]
\[ .3 \times .2 = .06 \]
\[ (.03)^2 = .0009 \]
\[ 3.21 \times 10 = 30.1 \times 10^2 = \]
\[ .01 \times 2 = .02 \]

Dividing decimals: change the problem to an equivalent whole number problem by multiplying both by the same power of ten.

Example:
\[ .3 \div .03 = \]
Multiply both by 10 to get
\[ .3 \div 3 = \]
\[ .010 \times .03 = .0003 \]
Multiply both by 1000 to get
\[ .3 \div 30 = \]
\[ .010 \times .003 = .00003 \]

G. Positive Integer Exponents and Square Roots of Perfect Squares

Meaning of exponents (powers):

Example:
\[ 3^2 = 3 \times 3 = 9 \]
\[ 3^3 = 3 \times 3 \times 3 = 27 \]

35 to 44: Find the value:
35. \[ 3^2 = \]
36. \[ (-3)^2 = \]
37. \[ (-3)^3 = \]
38. \[ (-3)^2 = \]
39. \[ (-3)^3 = \]

\[ \sqrt{a} \text{ is a non-negative real number if } a \geq 0 \]

\[ \sqrt{a} = b \text{ means } b^2 = a, \text{ where } b \geq 0. \text{ Thus } \sqrt{49} = 7, \text{ because } 7^2 = 49. \]
\[ \text{Also, } \sqrt{-49} = \text{ not defined.} \]

45. \[ \sqrt{1.44} = \]
46. \[ \sqrt{1.44} = \]
47. \[ \sqrt{-1.44} = \]
48. \[ \sqrt{1.44} = \]
49. \[ \sqrt{8100} = \]
D. Fraction-decimal conversion

Fraction to decimal: divide the top by the bottom.

\[ \text{example: } \frac{3}{4} = 3 \div 4 = 0.75 \]
\[ \text{example: } \frac{20}{3} = 20 \div 3 = 6.666666... = 6.\overline{6} \]
\[ \text{example: } \frac{32}{8} = 3 + \frac{2}{8} = 3 + 0.25 = 3.25 \]

52 to 55: Write each as a decimal. If the decimal repeats, show the repeating block of digits:

52. \[ \frac{5}{2} = 25 \text{ or } 2.5 \]
53. \[ \frac{3}{7} = 0.428571... \]

Non-repeating decimals to fractions: read the number as a fraction; write it as a fraction, reduce if possible:

\[ \text{example: } 0.6 = \frac{6}{10} = \frac{3}{5} \]
\[ \text{example: } 3.76 = \frac{376}{100} = \frac{94}{25} \]

56 to 58: Write as a fraction:

56. \[ \frac{3}{4} \]
57. \[ \frac{4}{9} = \frac{58}{1.25} \]

E. Percent

Meaning of percent: translate "percent" as "hundredths."

\[ \text{example: } 5\% = \frac{5}{100} = 0.05 \]

To change a decimal to percent form, multiply by 100; move the point 2 places right and write the percent symbol (%).

\[ \text{example: } 0.75 = 75\% \]
\[ \text{example: } 0.125 = 12.5\% \]

59 to 60: Write as a percent:

59. \[ \frac{1}{3} = 0.333333... = 33\frac{1}{3}\% \]

To change a percent to decimal form, move the point 2 places left and drop the % symbol.

\[ \text{example: } 8.76\% = 0.0876 \]
\[ \text{example: } 67\% = 0.67 \]

61 to 62: Write as a decimal:

61. \[ 10\% = 0.1 \]
62. \[ 0.03 = 0.03 \]

To solve a percent problem which can be written in this form:

\[ \text{a}\% \times \text{b} = \text{c} \]

First identify a, b, c:

63) to 65: If each statement were written (with the same meaning) in the form a% of b is c, identify a, b, and c:

63. \[ \text{3% of 20 is 1.2} \]
64. \[ \text{600 is 150% of 400} \]
65. \[ \text{3 out of 12 is 25%} \]

\[ \text{Given } a \text{ and } b, \text{ change } a\% \text{ to decimal form and multiply (since \textbf{of} can be translated \textbf{\textit{multiply}}).} \]

\[ \text{example: } \text{What is } 9.4\% \times 5000? \]
\[ (a\% \text{ of } b \text{ is: } \frac{9.4}{100} \times 5000 = 470) \]
\[ \text{example: } 56 \text{ problems right out of } 80 \text{ is what percent?} \]
\[ (a\% \text{ of } b \text{ is: } \frac{56}{80} \times 100 = 70\%) \]

66. \[ \frac{3}{4} \text{ of 9 is what?} \]
67. \[ \text{What percent of } 70 \text{ is } 56? \]
68. \[ 15\% \text{ of } 60 \text{ is what?} \]

F. Estimation and approximation

Rounding to one significant digit:

\[ \text{example: } 1.67 \text{ rounds to } 2 \]
\[ \text{example: } 0.049 \text{ rounds to } 0.0 \]
\[ \text{example: } 850 \text{ rounds to either } 800 \text{ or } 900 \]

69 to 71: Round to one significant digit:

69. \[ 1.5 \]
70. \[ 1.0 \]
71. \[ .00083 \]

To estimate an answer, an estimate is often sufficient to round each given number to one significant digit, then compute:

\[ \text{example: } 0.0298 \times 0.00513 \]
\[ \text{Round and compute: } 0.03 \times 0.0050 = 0.0000 \]
\[ 0.0005 \text{ is the estimate} \]

72 to 75: Select the best approximation of the answer:

72. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
73. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
74. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
75. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
76. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
77. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
78. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
79. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
80. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
81. \[ 1.23456 \times 367.00000 = 443,400,400,4000 \]
A. Grouping to simplify polynomials

The distributive property says \(a(b + c) = ab + ac\)

\[
\begin{align*}
\text{example:} & \quad 3(x - y) = 3x - 3y \\
& \quad (a = 3, b = x, c = -y) \\
\text{example:} & \quad 4x + 7x = 11x \\
& \quad (a = x, b = 4, c = 7) \\
\text{example:} & \quad 4a + 6x - 2 = 2(2a + 3x - 1) \\
\end{align*}
\]

1 to 3: Rewrite, using the distributive property:
1. \(6(x - 3) = -\)
2. \(4x - x = -\)
3. \(-5(a - 1) = -\)

Commutative and associative properties are also used in regrouping:

\[
\begin{align*}
\text{example:} & \quad 3x + 7 - x = 2x + 7 \\
\text{example:} & \quad 5 - x + 5 = 10 - x \\
\text{example:} & \quad 3x + 2y - 2x + 3y = 5x + 5y \\
\end{align*}
\]

4 to 9: Simplify:
4. \(x + x = 2x\)
5. \(a + b - a + b = 2b\)
6. \(9x - y + 3y = 8x + 2y\)
7. \(4x + 1 + x - 2 = 5x - 1\)
8. \(180 - x - 90 = 90 - x\)
9. \(x - 2y + y - 2x = -x - y\)

B. Evaluation by substitution

\[
\begin{align*}
\text{example:} & \quad \text{If } x = 3, \text{ then} \\
& \quad 7 - 4x = 7 - 4(3) \\
& \quad = 7 - 12 = -5 \\
\text{example:} & \quad \text{If } a = -7 \text{ and} \\
& \quad b = -1, \text{ then} \ a^2b = \\
& \quad (-7)^2(-1) = 49(-1) = -49 \\
\text{example:} & \quad \text{If } x = -2, \text{ then} \\
& \quad 3x^2 = x - 5 = \\
& \quad 3(-2)^2 = -2 - 5 = \\
& \quad 3 + 4 + 2 - 5 = 12 + 2 - 5 = \\
& \quad 9 \\
\end{align*}
\]

10 to 19: Given \(x = -1\), \(y = 3\), \(z = -3\), find the value:

10. \(2x^2 + 17 = 17 + x^2 + z = 13\)
11. \(-x^2z = -1\)
12. \(xz = 19\)
13. \(y + z = -4\)
14. \(y^2 + z^2 = 4 + 9 = 13\)
15. \(2x + 4y = -2 + 12 = 10\)
16. \(2x^2 - x - 1 = 2(1) - 1 = 1\)

C. Adding, subtracting polynomials

Combine like terms:

\[
\begin{align*}
\text{example:} & \quad (3x^2 + x + 1) - (x - 1) = 3x^2 + x + 1 - x + 1 = 3x^2 + 2 \\
\text{example:} & \quad (x - 1) + (x^2 + 2x - 3) = x - 1 + x^2 + 2x - 3 = x^2 + 3x - 4 \\
\text{example:} & \quad (a^2 + x - 1) - (6x^2 + 2x + 1) = a^2 + x - 1 - 6x^2 - 2x - 1 = -5x^2 + 3x - 2 \\
\end{align*}
\]

20 to 25: Simplify:

20. \((x^2 + x) - (x + 1) = x - 1\)
21. \((x - 3) + (5 - 2x) = -x + 2\)
22. \((a^2 - a) + (a^2 + a - 1) = 2a^2 - 1\)
23. \((y^2 - 3y - 5) = (2y^2 - y + 5)\)
24. \((7 - x) - (x - 7) = 14 - 2x\)
25. \(x^2 - (x^2 + x - 1) = -x + 1\)

D. Monomial times polynomial

Use the distributive property:

\[
\begin{align*}
\text{example:} & \quad 3(x - 4) = 3x - 12 \\
& \quad 3(-x) + 3(-4) = -3x - 12 \\
\text{example:} & \quad (2x + 3)(a) = 2ax + 3a \\
\text{example:} & \quad -4x(x^2 - 1) = -4x^3 + 4x \\
\end{align*}
\]

26. \(-x - 7\)
27. \(-2(3 - a) = -6 + 2a\)
28. \(x(x + 5) = x^2 + 5x\)
29. \((3x - 1)(x) = 3x^2 - x\)
30. \(a(2x - 3) = 2ax - 3a\)
31. \((x^2 - 1)(-1) = -x^2 + 1\)
32. \(3(2a^2 + 2a - 7) = 6a^2 + 6a - 21\)

E. Multiplying polynomials: use the distributive property:

\(a(b + c) = ab + ac\)

\[
\begin{align*}
\text{example:} & \quad (2x + 1)(x - 4) = 2x^2 + 2x - 8x - 4 \\
\text{example:} & \quad (x - 1)(x + 2) = x^2 + 2x - x - 2 = x^2 + x - 2 \\
\end{align*}
\]

Short cut to multiply above two binomials: FOIL (do mentally and write answer):

F: First times First:
\((2x)(x) = 2x^2\)

O: multiply 'Outers':
\((2x)(-4) = -8x\)

I: multiply 'Inners':
\((1)(x) = x\)

L: Last times Last:
\((1)(-4) = -4\)

Add, get \(2x^2 - 7x - 4\)
33 to 41: Multiply:
33. \((x + 3)^2 = \)
34. \((x - 3)^2 = \)
35. \((x + 3)(x - 3) = \)
36. \((2x + 3)(2x - 3) = \)
37. \((x - 4)(x - 2) = \)
38. \(-6x(x - 3) = \)
39. \((x - \frac{1}{2})^2 = \)
40. \((x - 1)(x + 3) = \)
41. \((x^2 - 1)(x^2 + 3) = \)

42 to 44: Match each pattern with its example:
42. I:  
43. II:  
44. III: 

45 to 52: Write the answer using the appropriate product pattern:
45. \((3a + 1)(3a - 1) = \)
46. \((y - 1)^2 = \)
47. \((3a + 2)^2 = \)
48. \((3a + 2)(3a - 2) = \)
49. \((3a - 2)(3a - 2) = \)
50. \((x - y)^2 = \)
51. \((lx + my)^2 = \)
52. \((3x + y)(3x - y) = \)

4. Factoring
Monomial factors:
ab + ac = a(b + c)

Examples:
\[x^2 - x = x(x - 1)\]
\[lx^2 + 6xy = 2xy(2x + 3)\]

Difference of two squares:
\[a^2 - b^2 = (a + b)(a - b)\]

Example:
\[9x^2 - 1 = (3x + 2)(3x - 2)\]

Trinomial square:
\[a^2 + 2ab + b^2 = (a + b)^2\]
\[a^2 - 2ab + b^2 = (a - b)^2\]

Example:
\[x^2 - 6x + 9 = (x - 3)^2\]

3 to 10: Special products
These product patterns (examples of FOIL) should be remembered and recognized:
I. \((a + b)(a - b) = a^2 - b^2\)
II. \((a + b)^2 = a^2 + 2ab + b^2\)
III. \((a - b)^2 = a^2 - 2ab + b^2\)

Example 1:
\[(3x - 1)^2 = 9x^2 - 6x + 1\]

Example 2:
\[(x + 5)^2 = x^2 + 10x + 25\]

Example 3:
\[(x + 8)(x - 8) = x^2 - 64\]

53 to 67: Factor:
53. \(a^2 + ab = \)
54. \(a^3 - a^2b + ab^2 = \)
55. \(8x^2 - 2 = \)
56. \(x^2 - 10x + 25 = \)
57. \(-6xy + 10x^2 = \)
58. \(2x^2 - 3x - 5 = \)
59. \(x^2 = x - 6 = \)
60. \(x^2 = y^2 = \)
61. \(x^2 = y^2 = \)
62. \(x^2 = y^2 = \)
63. \(x^3 + 8x^2 + 2x = \)
64. \(9x^2 + 12x + 4 = \)
65. \(6x^2 - 9x + y = \)
66. \(1 - x - 2x^2 = \)
67. \(3x^2 = 10x + 3 = \)
A. Solving one linear equation in one variable: add or subtract the same thing on each side of the equation, or multiply or divide each side by the same thing, with the goal of getting the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

1 to 11: Solve:

1. \(2x = 9\)  
   \(x = \frac{9}{2}\)

2. \(3 = \frac{6x}{5}\)  
   \(x = \frac{5}{2}\)

3. \(3x + 7 = 6\)  
   \(x = \frac{1}{3}\)

4. \(\frac{x}{4} = \frac{5}{4}\)  
   \(x = 5\)

5. \(5 - x = 9\)  
   \(x = -4\)

6. \(x = \frac{2x}{2} + 1\)  
   \(x = 3\)

7. \(4x - 6 = x\)  
   \(x = 2\)

8. \(x - 4 = \frac{2}{x} + 1\)  
   \(x = 2\)

9. \(6 - 4x = x\)  
   \(x = 1\)

10. \(7x - 5 = 2x + 10\)  
    \(x = 5\)

11. \(4x + 5 = 3 - 2x\)  
    \(x = -\frac{1}{3}\)

To solve a linear equation for one variable in terms of the other(s), do the same as above:

example: Solve for \(P: C = \frac{6}{5}(P - 32)\)

Multiply by \(\frac{5}{6}\):  
\(\frac{5}{6}C = P - 32\)

Add 32:  
\(P = \frac{5}{6}C + 32\)

Thus, \(P = \frac{5}{6}C + 32\)

example: Solve for \(b: a + b = 90\)

Subtract \(a\):  
\(b = 90 - a\)

example: Solve for \(x: ax + b = 0\)

Subtract \(b\):  
\(ax = -b\)

Divide by \(a\):  
\(x = \frac{-b}{a}\)

12 to 19: Solve for the indicated variable in terms of the other(s):

12. \(3a + b = 180\) \(b = \frac{180 - b}{3}\)

13. \(2a + 2b = 180\) \(b = \frac{180 - 2a}{2}\)

14. \(P = 2b + 2h\) \(b = \frac{P - 2h}{2}\)

15. \(y = 3x - 2\) \(x = \frac{y + 2}{3}\)

B. Solution of a one-variable equation reducible to a linear equation: some equations which don't appear linear can be solved by using a related linear equation:

example: \(\frac{x + 1}{x} = -1\)

Multiply by \(x\):  
\(x + 1 = -x\)

Solve:

\(2x = -1\)

\(x = -\frac{1}{2}\)

(If you get a check answer in the original equation.)

example: \(\frac{2x + 1}{x + 1} = 5\)

Think of \(5\) as \(\frac{5}{1}\) and

cross-multiply:

\(5x + 5 = 3x + 3\)

\(2x = -2\)

\(x = -1\)

But \(x = -1\) doesn't make the original equation true (doesn't check), so there is no solution.

20 to 25: Solve and check:

20. \(\frac{x - 1}{x + 1} = \frac{6}{7}\)

21. \(\frac{3x}{2x + 1} = \frac{5}{3}\)

22. \(\frac{3x - 2}{2x + 1} = 4\)

23. \(\frac{x + 1}{2x} = 2\)

24. \(\frac{1}{3} = \frac{x}{x + 6}\)

25. \(\frac{x - 2}{4} = \frac{2x}{2x - 3}\)

example: \(|3 - x| = 2\)

Since the absolute value of both 2 and -2 is 2, \(3 - x\) can be either 2 or -2. Write these two equations and solve each:

\(3 - x = 2\) or \(3 - x = -2\)

\(-x = -1\) or \(-x = -5\)

\(x = 1\) or \(x = 5\)

26 to 30: Solve:

26. \(|x| = 3\)

27. \(|x| = -1\)

28. \(|x - 1| = 3\)

29. \(|2 - 3x| = 0\)

30. \(|x + 2| = 1\)
C. Solution of linear inequalities

Rules for inequalities:

<table>
<thead>
<tr>
<th>If ( a &gt; b ), then:</th>
<th>If ( a &lt; b ), then:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a + c &gt; b + c )</td>
<td>( a + c &lt; b + c )</td>
</tr>
<tr>
<td>( a - c &gt; b - c )</td>
<td>( a - c &lt; b - c )</td>
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<tr>
<td>( ac &gt; bc ) (if ( c &gt; 0 ))</td>
<td>( ac &lt; bc ) (if ( c &gt; 0 ))</td>
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<td>( ac &gt; bc ) (if ( c &lt; 0 ))</td>
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<td>( \frac{a}{c} &gt; \frac{b}{c} ) (if ( c &gt; 0 ))</td>
<td>( \frac{a}{c} &lt; \frac{b}{c} ) (if ( c &gt; 0 ))</td>
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</tr>
</tbody>
</table>

Example: One variable graph: solve and graph on number line: \( 1 \leq 2x \leq 7 \)
(This is an abbreviation for \( \{x : 1 \leq 2x \leq 7\} \)
Subtract 1, get \(-2x \leq 6\)
Divide by \(-2\), \(x \geq -3\)
Graph: \([-4, -3, -2, -1, 0, 1, 2, 3]\)

31 to 38: Solve and graph on number line:
31. \( x - 3 > 4 \) 35. \( 4 - 2x < 6 \)
32. \( 4x < 2 \) 36. \( 5 - x > x - 3 \)
33. \( 2x + 1 \leq 6 \) 37. \( x > 1 + 4 \)
34. \( 3 < x - 3 \) 38. \( 6x + 5 \geq 4x - 3 \)

D. Solving a pair of linear equations
in two variables: the solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

39 to 46: Solve for the common solution(s) by substitution or linear combinations:
39. \( x + 2y = 7 \) 43. \( 2x - 3y = 5 \)
\( 3x - y = 28 \) 3x + 5y = 1
40. \( x + y = 5 \) 44. \( 4x - 1 = y \)
\( x - y = -3 \) 4x + y = 1
41. \( 2x - y = -9 \) 45. \( x + y = 3 \)
\( x = 8 \) \( x + y = 1 \)
42. \( 2x - y = 1 \) 46. \( 2x - y = 3 \)
\( y = x - 5 \) 6x - 9 = 3y

Answers:
1. \( \frac{9}{2} \)
2. \( \frac{5}{2} \)
3. \(-1/3\)
4. \( 15/4 \)
5. \(-4 \)
6. \( 5/3 \)
7. \( 2 \)
8. \( 10 \)
9. \( 6/5 \)
10. \( 3 \)
11. \(-1/3\)
12. \( 180 - a \)
13. \( 90 - a \)
14. \( (7 - 2a)/2 \)
15. \( (y + 21)/3 \)
16. \( 4 - y \)
17. \( (3y - 3)/2 \)
18. \(-by/a\)
19. \( x/b \)
20. \( 13 \)
21. \(-5/4 \)
22. \(-6/5 \)
23. \( 1 \)
24. \( h \)
25. no solution
26. \(-3, 3\)
27. no solution
28. \(-2, 4\)
29. \(2/3\)
30. \(-3, -1\)
31. \( x \geq 7 \)
32. \( x < 1/2 \)
33. \( x \leq 5/2 \)
34. \( x < 6 \)
35. \( x = -1 \)
36. \( x > 4 \)
37. \( x = 5 \)
38. \( x \leq 4 \)
39. \((-9, -1)\)
40. \((1, 6)\)
41. \((8, 28)\)
42. \((-4, -9)\)
43. \((28/29, -1/19)\)
44. \((1/4, 0)\)
45. no solution
46. any ordered pair of the form \((a, 28 - 3)\)
where \(a\) is any number. One example: \((1, 5)\).
Infinitely many solutions.
Elementary Algebra Diagnostic Test Practice

Topic 4: Quadratic equations

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. $ax^2 + bx + c = 0$: a quadratic equation can always be written so it looks like

$ax^2 + bx + c = 0$

where $a$, $b$, and $c$ are real numbers and $a$ is not zero.

Example: $5 - x = 3x^2$

Add $x$: $5 = 3x^2 + x$

Subtract 5: $0 = 3x^2 + x - 5$

or $3x^2 + x - 5 = 0$

So $a = 3$, $b = 1$, $c = -5$

Example: $x^2 = 3$

Rewrite: $x^2 - 3 = 0$

(Think of $x^2 + 0x - 3 = 0$)

So $a = 1$, $b = 0$, $c = -3$

1 to 4: Write each of the following in the form $ax^2 + bx + c = 0$ and identify $a$, $b$, $c$:

1. $3x + x^2 - 4 = 0$
2. $5 - x^2 = 0$
3. $x^2 = 3x - 1$
4. $x = 3x^2$
5. $6x^2 = 1$

B. Factoring

Monomial factors:

$ab + ac = a(b + c)$

Examples:

$x^2 - x = x(x - 1)$

$4x^2y + 6xy = 2xy(2x + 3)$

Difference of two squares:

$a^2 - b^2 = (a + b)(a - b)$

Example:

$9x^2 - 4 = (3x + 2)(3x - 2)$

Trinomial square:

$a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

Example:

$x^2 - 6x + 9 = (x - 3)^2$

Trinomial:

Examples:

$x^2 - x - 2 = (x - 2)(x + 1)$

$6x^2 - 7x - 3 = (3x + 1)(2x - 3)$

6 to 20: Factor:

6. $a^2 + ab =$
7. $a^3 - a^2b + ab^2 =$
8. $8x^2 - 2 =$
9. $x^2 - 10x + 25 =$
10. $-4xy + 10x^2 =$
11. $2x^2 - 3x - 5 =$
12. $x^2 - x - 6 =$
13. $x^2 - y^2x =$
14. $x^2 - 3x - 10 =$
15. $2x^2 - x =$
16. $2x^3 + 8x^2 + 8x =$
17. $9x^2 + 12x + 4 =$
18. $6x^2y - 9xy =$
19. $1 - x = 2x^2 =$
20. $3x^2 - 10x + 3 =$

C. Solving factored quadratic equations: the following statement is the central principle:

If $ab = 0$,
then $a = 0$ or $b = 0$

First, identify $a$ and $b$ in $ab = 0$:

Example: $(3 - x)(x + 2) = 0$

Compare this with $ab = 0$

$a = (3 - x)$

$b = (x + 2)$

21 to 24: Identify $a$ and $b$ in each of the following:

21. $3x(2x - 5) = 0$
22. $(x - 3)x = 0$
23. $(2x - 1)(x - 5) = 0$
24. $0 = (x - 1)(x + 1)$

Then, because $ab = 0$ means $a = 0$ or $b = 0$, we can use the factors to make two linear equations to solve:

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One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied by the Mathematics Diagnostic Testing Project, Gavilan Center Suite 704, U.S.A. 905 Hillard Ave., Los Angeles, CA 90024.
example: if \(2x(3x - 4) = 0\)
then \((2x) = 0\) or \((3x - 4) = 0\)
and so \(x = 0\) or \(3x = 4\)
\(x = \frac{4}{3}\)
Thus, there are two solutions: \(0\) and \(\frac{4}{3}\)

example: if \((3 - x)(x + 2) = 0\)
then \((3 - x) = 0\) or \((x + 2) = 0\)
and thus \(x = 3\) or \(x = -2\)

example: if \((2x + 7)^2 = 0\)
then \(2x + 7 = 0\)
\(2x = -7\)
\(x = -\frac{7}{2}\) (one solution)

Note: there must be a zero on one side of the equation to solve by the factoring method.

25 to 31: Solve:
25. \((x + 1)(x - 1) = 0\)
26. \(4x(x + 4) = 0\)
27. \(0 = (2x - 5)x\)
28. \(0 = (2x + 3)(x - 1)\)
29. \((x - 6)(x - 6) = 0\)
30. \((2x - 3)^2 = 0\)
31. \(x(x + 2)(x - 3) = 0\)

D. Solving quadratic equations by factoring: arrange the equation so zero is on one side (in the form \(ax^2 + bx + c = 0\)), factor, set each factor equal to zero, and solve the resulting linear equations.

example: solve \(6x^2 = 3x\)
Rewrite: \(6x^2 - 3x = 0\)
Factor: \(3x(2x - 1) = 0\)
So \(3x = 0\) or \((2x - 1) = 0\)
Thus \(x = 0\) or \(x = \frac{1}{2}\)

example: \(x = x - 4\) or \(x + 3 = 0\)
\(x = 4\) or \(x = -3\)

32 to 43: Solve by factoring:
32. \(x(x - 3) = 0\)
33. \(x^2 - 2x = 0\)
34. \(2x^2 = x\)
35. \(3x(x + 4) = 0\)
36. \(x^2 = 2 - x\)
37. \(x^2 + x = 6\)
38. \(0 = (x + 2)(x - 3)\)
39. \((2x + 1)(3x - 2) = 0\)

40. \(6x^2 = x + 2\)
41. \(9 + x^2 = 6x\)
42. \(1 - x = 2x^2\)
43. \(x^2 - x - 6 = 0\)

Another problem form: if a problem is stated in this form: 'One of the solutions of \(ax^2 + bx + c = 0\) is \(d\)', solve the equation as above, then verify the statement.

example: Problem: One of the solutions of \(10x^2 - 5x = 0\) is
A. \(-2\)
B. \(-1/2\)
C. \(1/2\)
D. \(2\)
E. \(5\)

Solve \(10x^2 - 5x = 0\) by factoring: \(5x(2x - 1) = 0\)
so \(5x = 0\) or \(2x - 1 = 0\)
thus \(x = 0\) or \(x = 1/2\)
Since \(x = 1/2\) is one solution,
answer C is correct.

44. One of the solutions of \((x - 1)(3x + 2) = 0\) is
A. \(-3/2\)
B. \(-2/3\)
C. \(0\)
D. \(2/3\)
E. \(3/2\)

45. One solution of \(x^2 - x - 2 = 0\) is
A. \(-2\)
B. \(-1\)
C. \(-1/2\)
D. \(1/2\)
E. \(1\)

Annex:

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Elementary Algebra Diagnostic Test Practice

Topic 5: Graphing

Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Graphing a point on the number line

1. To 7: Select the letter of the point on the number line with coordinate:

   0 1 2 3 4 5 6 7
   A B C D E F G
   -2 -1 0 1 2 3

   1. 0
   2. 1
   3. -1/2
   4. 1/3
   5. 1
   6. 2.75
   7. -2

8. Which letter best locates the given number:

   0 1/2 1 3/2 2 3/2 3 1
   A B C D E F G H

   8. 5/4

9. 1/2

11 to 13: Solve each equation and graph the solution on the number line:

   Example: x + 3 = 1
   x = -2
   -3 -2 -1 0 1

   11. 2x - 6 = 0
   12. x = 3x + 5
   13. 4 - x = 3 + x

B. Graphing a linear inequality in one variable on the number line

Rules for inequalities:

If a > b, then: If a < b, then:

a + c > b + c a + c < b + c
a - c > b - c a - c < b - c
ac > bc (if c>0) ac < bc (if c<0)
ac < bc (if c<0) ac > bc (if c<0)
a > b (if c>0) a < b (if c>0)
\overline{a} < \overline{b} (if c<0)
\overline{a} > \overline{b} (if c<0)

Example: One variable graph: solve and graph on number line: 1 - 2x ≤ 7
(This is an abbreviation for [x: 1 - 2x ≤ 7])
Subtract 1, get -2x ≤ 6
Divide by -2, x ≥ -3
Graph: -4 -3 -2 -1 0 1 2 3

14 to 20: Solve and graph on number line:

14. x - 3 > 4
15. 4x < 2
16. 2x + 1 ≤ 6
17. 3 < x - 3
18. 4 - 2x < 6
19. 5 - x > x - 3
20. x + 1 ≤ 4

C. Graphing a point in the coordinate plane

If two number lines intersect at right angles so that:

1) one is horizontal with positive to the right and negative to the left,
2) the other is vertical with positive up and negative down, and
3) the zero points coincide, then they form a coordinate plane, and
1) the horizontal number line is called the x-axis,
2) the vertical line is the y-axis,
3) the common zero point is the origin,
4) there are four quadrants, numbered as shown:

To locate a point on the plane, an ordered pair of numbers is used, written in the form (x, y). The x-coordinate is always given first.
Identify $x$ and $y$ in each ordered pair:

24. $(3, 0)$  
25. $(-2, 5)$  
26. $(5, -2)$  
27. $(0, 3)$

To plot a point, start at the origin and make the two moves, first in the $x$-direction (horizontal) and then in the $y$-direction (vertical) indicated by the ordered pair.

**Example:** $(-3, 4)$

- Start at the origin, move left 3 (since $x = -3$),
- then (from there), up 4 (since $y = 4$).
- Put a dot there to indicate the point $(-3, 4)$.

28. Join the following points in the given order:
- $(3, 0)$, $(0, 0)$, $(-3, 0)$,
- $(0, 3)$, $(0, -3)$.

29. Two of the lines you drew cross each other. What are the coordinates of this crossing point?

30. In what quadrant does the point $(a, b)$ lie, if $a > 0$ and $b < 0$?

31 to 34: For each given point, which of its coordinates, $x$ or $y$, is larger?

35 to 41: Graph each line on the number plane and find its slope (refer to section E below if necessary):

- $y = 3x - 1$
- $x - y = 3$
- $x = 1 - y$
- $y = 1$

E. Slope of a line through two points

42 to 47: Find the value of each of the following:

- $y = 3x - 1$
- $x - y = 3$
- $x = 1 - y$
- $y = 1$

48. $A(3, -1)$, $B(-2, 4)$

- Find the slope of $\overline{AB}$.
- And $P_2(x_2, y_2)$ has slope $\frac{y_2 - y_1}{x_2 - x_1}$.

48. $A(3, -1)$, $B(-2, 4)$
- Find the line joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ has slope $\frac{y_2 - y_1}{x_2 - x_1}$.

Answers:

1. $y = 2x + 1$
2. $y = 0$
3. $y = 3$
4. $y = 3$
5. $y = 3$
6. $y = 3$
7. $y = 3$
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19. $y = 3$
20. $y = 3$
21. $y = 3$
22. $x = 2$
23. $1 < x < 4$
24. $3$
25. $2$
26. $1$
27. $0$
28. $1$
29. $(-1, 1)$
30. $(-1, 1)$
31. $x$
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Elementary Algebra Diagnostic Test Practice
Topic 6: Rational Expressions
Directions: Study the examples, work the problems, then check your answers on the back of this sheet. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

A. Simplifying fractional expressions

Example: $\frac{\frac{2}{4}}{\frac{2}{3} + \frac{1}{3}} = \frac{\frac{2}{4} \cdot \frac{3}{3}}{\frac{2}{3} \cdot \frac{3}{3}} = \frac{\frac{2}{4}}{1} = \frac{2}{4} = \frac{3}{4}$ (note that you must be able to find a common factor—in this case 9—in both the top and bottom in order to reduce a fraction.)

Example: $\frac{3a}{12ab} = \frac{3a}{12b} = \frac{3a}{4b} \cdot \frac{1}{4} = \frac{1}{4b}$

(common factor: 3a)

1 to 12: Reduce:

1. $\frac{12}{24} = \frac{1}{2}$
2. $\frac{26}{52} = \frac{1}{2}$
3. $\frac{3}{9} + \frac{6}{9} = \frac{3}{3}$
4. $\frac{6a}{15b} = \frac{2a}{5b}$
5. $\frac{19a^2}{55a} = \frac{19a}{55}$
6. $\frac{14x - 7y}{14y} = \frac{x}{y}$
7. $\frac{2a + b}{5a + c} = \frac{2}{5}$
8. $\frac{x + 4}{x - 4}
9. \frac{2(x + 4)(x - 5)}{(x - 5)(x - 4)} = \frac{2x + 2}{x - 4}$
10. $\frac{x^2 - 2x}{x - 9}$
11. $\frac{8(x + 1)^2}{6(x^2 - 1)}$
12. $\frac{2x^2 - x - 1}{x^2 - 2x + 1}$

Example: $\frac{1}{x} \cdot \frac{y}{10x} = \frac{1 \cdot 2y}{1 \cdot 2x} = \frac{1}{x} \cdot \frac{1}{y}$

13 to 14: Simplify:

13. $\frac{4x}{6} \cdot \frac{3y}{2} = 2x\cdot 2y$
14. $\frac{x^2 - 3x}{x - 4} \cdot \frac{2(x - 4)}{2x - 6}$

B. Evaluation of fractions

Example: If $a = -1$ and $b = 2$, find the value of $\frac{a + 3}{2b - 1}$
Substitute: $\frac{-1 + 3}{2(2) - 1} = \frac{2}{3}$

15 to 22: Find the value, given $a = -1$, $b = 2$, $c = 0$, $x = -3$, $y = 1$, $z = 2$:

15. $\frac{6}{5}$
16. $\frac{x}{y}$
17. $\frac{x}{3}$
18. $\frac{a - y}{b}$
19. $\frac{4x - 5x}{3y - 2x}$
20. $\frac{c}{a}$
21. $\frac{a}{b}$
22. $\frac{6}{x}$

C. Equivalent fractions

Example: $\frac{3}{4}$ is equivalent to how many eights? $\frac{3}{4} \cdot \frac{8}{8} = \frac{24}{32}$

Example: $\frac{6}{5a} = \frac{5ab}{5ab}$

Example: $\frac{2x + 2}{x + 1} = \frac{4(x + 1)}{2x + 2}$

Example: $\frac{x - 1}{x + 1} = \frac{(x - 1)(x + 1)}{x^2 - 1}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

Example: $\frac{5}{6}$ and $\frac{8}{15}$

First find LCM of 6 and 15:
$6 = 2 \cdot 3$
$15 = 3 \cdot 5$
LCM = $2 \cdot 3 \cdot 5 = 30$, so
$\frac{5}{6} = \frac{25}{30}$, and $\frac{8}{15} = \frac{16}{30}$

Example: $\frac{1}{4}$ and $\frac{1}{6a}$

Example: $\frac{3}{x + 2}$ and $\frac{-1}{x - 2}$

LCM = $(x + 2)(x - 2)$, so
$x + 2 = \frac{3}{x + 2}(x + 2)(x - 2)$
$-1 = \frac{3}{x - 2}(x + 2)(x - 2)$

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D. Adding and subtracting fractions: if denominators are the same, combine the numerators:

*Example:* \( \frac{2x}{3} - \frac{x}{3} = \frac{2x - x}{3} = \frac{2x}{3} \)

34. \( \frac{4}{7} + \frac{2}{7} = \frac{6}{7} \)
35. \( \frac{3}{5} - \frac{x}{5} = \frac{3 - x}{5} \)
36. \( \frac{b - a}{b + a} - \frac{a - b}{b + a} = \frac{b - a - (a - b)}{b + a} = \frac{2b}{2b + 2a} \)
37. \( \frac{x + 2}{x^2 + 2x} - \frac{2x}{x^2 + 2x} = 0 \)
38. \( \frac{3a}{b} + \frac{2}{b} - \frac{a}{b} = \frac{3a + 2 - a}{b} = \frac{2a + 2}{b} \)

If denominators are different, find equivalent fractions with common denominators, then proceed as before (combine numerators):

*Example:* \( \frac{a}{b} - \frac{a}{4} = \frac{4a - a}{4b} = \frac{3a}{4b} \)

*Example:* \( \frac{1}{x + 2} + \frac{1}{x + 2} = \frac{x - 1}{x + 2} + \frac{x - 1}{x + 2} = \frac{x - 1 + x - 1}{x + 2} = \frac{2x - 2}{x + 2} \)

39 to 51: Find the sum or difference:

39. \( \frac{3}{a} - \frac{3}{2a} = \frac{6}{a} - \frac{3}{2a} = \frac{6 - 3}{2a} = \frac{3}{2a} \)
40. \( \frac{3}{x} - \frac{2}{a} = \frac{3a - 2x}{2a} \)
41. \( \frac{4}{5} - \frac{2}{x} = \frac{4x - 2}{5x} \)
42. \( \frac{4}{5} + \frac{1}{a} = \frac{4a + 5}{5a} \)
43. \( \frac{1}{x + 1} + \frac{1}{x - 1} = \frac{x - 1 + x + 1}{x^2 - 1} = \frac{2x}{x^2 - 1} \)
44. \( \frac{2x - 2}{x + 2} = \frac{2(x - 1)}{x + 2} \)
45. \( \frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab} \)
46. \( \frac{3}{a} + \frac{1}{a} = \frac{4}{a} \)
47. \( \frac{3}{a} - \frac{2}{a^2} = \frac{3a - 2}{a^2} \)
48. \( \frac{2x - 2}{x^2} = \frac{2(x - 1)}{x^2} \)
49. \( \frac{2x}{x + 1} - \frac{2x}{x - 1} = \frac{2x^2 - 2x - 2x^2 + 2x}{x^2 - 1} = \frac{4}{x^2 - 1} \)
50. \( \frac{x}{x - 2} - \frac{4}{x - 2} = 0 \)
51. \( \frac{x}{x - 2} - \frac{4}{x^2 - 4} = \frac{x^2 - 4}{x(x - 2)} \)

E. Multiplying fractions: multiply the tops, multiply the bottoms, reduce if possible:

*Example:* \( \frac{2}{3} \cdot \frac{3}{5} = \frac{6}{15} = \frac{2}{5} \)

*Example:* \( \frac{1}{x + 1} \cdot \frac{x^2 - 4}{x - 2} \cdot \frac{(x - 1)(x - 2)}{(x - 1)(x + 1)} = \frac{2x^2 + 6}{x - 1} \)

52. \( \frac{2}{3} \cdot \frac{3}{5} = \frac{6}{15} = \frac{2}{5} \)
53. \( \frac{a}{b} \cdot \frac{b}{a} = \frac{a^2}{b^2} \)
54. \( \frac{2}{7} \cdot \frac{ab}{12} = \frac{2ab}{84} \)
55. \( \frac{2}{5} \cdot \frac{3}{ab} = \frac{6}{5ab} \)
56. \( \frac{a^3}{b^3} \cdot \frac{3}{5y} = \frac{3a^3}{5by} \)
57. \( \frac{2x^2 + 2x - 1}{2x + 3} \)
58. \( \frac{2x^2 + 2x - 1}{2x + 1} \cdot \frac{3y}{x^2 + 6} = \frac{6xy}{x^2 + 6} \)
59. \( \frac{x + 3}{3x} \cdot \frac{x^2}{2x} = \frac{x^3}{6x^2} = \frac{x}{6} \)

F. Dividing fractions: a nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

*Example:* \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \)

*Example:* \( \frac{7}{3} \div \frac{3}{5} = \frac{7 \cdot 5}{3 \cdot 3} = \frac{35}{9} = \frac{42}{1} = 42 \)

*Example:* \( \frac{5x}{2y} \div 2x = \frac{5x}{2xy} = \frac{5x}{2y} \cdot \frac{1}{2x} = \frac{5x}{2y} = \frac{5}{4y} \)

60. \( \frac{3}{4} \cdot \frac{2}{3} = \frac{6}{12} = \frac{1}{2} \)
61. \( \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \)
62. \( \frac{5}{6} \cdot 2 = \frac{10}{6} = \frac{5}{3} \)
63. \( \frac{2}{3} \cdot \frac{3}{2} = \frac{6}{6} = 1 \)
64. \( \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8} \)
65. \( \frac{8}{9} \div \frac{1}{2} = \frac{16}{9} \)
66. \( \frac{a}{b} \div \frac{b}{a} = \frac{a}{b} \cdot \frac{a}{b} = \frac{a^2}{b^2} \)

Answers:
1. \( \frac{1}{2} \)
2. \( \frac{3}{5} \)
3. \( \frac{3}{5} \)
4. \( \frac{3}{5} \)
5. \( \frac{1}{5} \)
6. \( \frac{1}{5} \)
7. \( \frac{1}{5} \)
8. \( \frac{1}{5} \)
9. \( \frac{1}{5} \)
10. \( x \)
11. \( \frac{1}{x - 1} \)
12. \( \frac{3}{x - 1} \)
13. \( \frac{3}{x - 1} \)
14. \( \frac{x - 1}{x^2} \)
15. \( \frac{3}{x - 1} \)
16. \( \frac{3}{x - 1} \)
17. \( \frac{1}{x - 1} \)
18. \( \frac{1}{x - 1} \)
19. \( \frac{1}{x - 1} \)
20. \( \frac{1}{x - 1} \)
21. \( \frac{1}{x - 1} \)
22. \( \frac{1}{x - 1} \)
23. \( \frac{1}{x - 1} \)
24. \( \frac{1}{x - 1} \)
25. \( \frac{1}{x - 1} \)
26. \( \frac{1}{x - 1} \)
27. \( \frac{1}{x - 1} \)
28. \( \frac{1}{x - 1} \)
29. \( \frac{1}{x - 1} \)
30. \( \frac{1}{x - 1} \)
31. \( \frac{1}{x - 1} \)
32. \( \frac{1}{x - 1} \)
33. \( \frac{1}{x - 1} \)
34. \( \frac{1}{x - 1} \)
35. \( \frac{1}{x - 1} \)
36. \( \frac{1}{x - 1} \)
37. \( \frac{1}{x - 1} \)
38. \( \frac{1}{x - 1} \)
39. \( \frac{1}{x - 1} \)
40. \( \frac{1}{x - 1} \)
41. \( \frac{1}{x - 1} \)
42. \( \frac{1}{x - 1} \)
43. \( \frac{1}{x - 1} \)
44. \( \frac{1}{x - 1} \)
45. \( \frac{1}{x - 1} \)
46. \( \frac{1}{x - 1} \)
47. \( \frac{1}{x - 1} \)
### Elementary Algebra Diagnostic Test Practice

**Topic 7: Exponents and square roots**

**Directions:** Study the examples, work the problems, then check your answers on the back of this sheet. If you don’t get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

#### A. Positive integer exponents

- **Example:** $25^2 \cdot 2 = 2^{2+2}$, and has value 32.

<table>
<thead>
<tr>
<th>1 to 14: Find the value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.  $2^3 = 8$</td>
</tr>
<tr>
<td>2.  $3^2 = 9$</td>
</tr>
<tr>
<td>3.  $-4^2 = -16$</td>
</tr>
<tr>
<td>4.  $(4^2)^2 = 4^4 = 256$</td>
</tr>
<tr>
<td>5.  $6^4 = 1296$</td>
</tr>
<tr>
<td>6.  $(-3)^2 = 9$</td>
</tr>
<tr>
<td>7.  $(\frac{2}{3})^4 = \frac{16}{81}$</td>
</tr>
<tr>
<td>8.  $(\frac{2}{3})^3 = \frac{8}{27}$</td>
</tr>
<tr>
<td>9.  $(\frac{1}{2})^2 = \frac{1}{4}$</td>
</tr>
<tr>
<td>10.  $2^{-10} = \frac{1}{1024}$</td>
</tr>
<tr>
<td>11.  $(-2)^9 = -512$</td>
</tr>
<tr>
<td>12.  $(2^{1/2})^2 = 2$</td>
</tr>
<tr>
<td>13.  $(-1.1)^3 = -1.331$</td>
</tr>
<tr>
<td>14.  $3^2 \cdot 2^3 = 36$</td>
</tr>
</tbody>
</table>

**Example:** Simplify: $a \cdot a \cdot a \cdot a = a^4$

<table>
<thead>
<tr>
<th>15 to 18: Simplify:</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.  $3^2 \cdot x^4$</td>
</tr>
<tr>
<td>16.  $2^4 \cdot b \cdot b \cdot b = 16b^3$</td>
</tr>
<tr>
<td>17.  $4^{(x-x)(x-x)} = 1$</td>
</tr>
<tr>
<td>18.  $(x^3)^4 = x^{12}$</td>
</tr>
</tbody>
</table>

#### B. Integer exponents

- **Example:** $2^{-3} = \frac{1}{8}$

| I.  $a^b \cdot a^c = a^{b+c}$ |
| II.  $\frac{a^b}{a^c} = a^{b-c}$ |
| III.  $(a^b)^c = a^{bc}$ |
| IV.  $(ab)^c = a^c \cdot b^c$ |
| V.  $a^{\frac{b}{c}} = \sqrt{a^b}$ |
| VI.  $a^0 = 1$ (if $a \neq 0$) |
| VII.  $a^{-b} = \frac{1}{a^b}$ |

#### C. Scientific notation

- **Example:** $32800 = 3.28 \times 10^4$

| 19.  $a^3 \cdot 2^4 = 2^x$ |
| 20.  $3^3 \cdot 2^4 \neq 2^x$ |
| 21.  $3^4 = \frac{1}{3^x}$ |
| 22.  $2^2 \cdot 5^2 = 2^x$ |
| 23.  $(2^3)^4 = 2^x$ |
| 24.  $8 = 2^x$ |
| 25.  $a^3 \cdot a = a^x$ |
| 26.  $b^{10} \cdot a = b^x$ |
| 27.  $\frac{1}{c^{10}} = c^x$ |
| 28.  $a^{3y} = 2$ |

**To compute with numbers written in scientific notation, separate the parts, compute, then recombine.**

- **Example:** $(3.14 \times 10^5)(2) = 6.28 \times 10^5$
- **Example:** $(4.28 \times 10^6)^2 = 2.20 \times 10^{12}$
- **Example:** $(2.01 \times 10^{-3}) = 8.04 \times 10^{-5}$

<table>
<thead>
<tr>
<th>29 to 41: Find the value:</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.  $7x^0 = 7$</td>
</tr>
<tr>
<td>30.  $3^4 = 81$</td>
</tr>
<tr>
<td>31.  $2^3 \cdot 2^4 = 2^7$</td>
</tr>
<tr>
<td>32.  $0^5 = 0$</td>
</tr>
<tr>
<td>33.  $5^0 = 1$</td>
</tr>
<tr>
<td>34.  $(-3)^3 = -27$</td>
</tr>
<tr>
<td>35.  $x^0 + 3 \cdot x^{-3} = 2$</td>
</tr>
<tr>
<td>36.  $x^3 \div x^0 = 1$</td>
</tr>
<tr>
<td>37.  $2x^3 \div 6x^4 = \frac{1}{3}$</td>
</tr>
<tr>
<td>38.  $(a^x + a^{-3})(x-3) = 2x$</td>
</tr>
<tr>
<td>39.  $(x^3)^2 = x^6$</td>
</tr>
<tr>
<td>40.  $(3x^2)^2 = 9x^4$</td>
</tr>
<tr>
<td>41.  $(-2xy^2)^3 = -8x^3y^6$</td>
</tr>
</tbody>
</table>

**Note that scientific form always looks like $a \times 10^n$ where $1 \leq a < 10$, and $n$ is an integer power of 10.**

- **42.** $93,000,000 = 9.3 \times 10^7$
- **43.** $0.000042 = 4.2 \times 10^{-5}$
- **44.** $5.07 = 5.07 \times 10^0$
- **45.** $-32 = -3.2 \times 10^1$

- **46.** $1.403 \times 10^3 = 1403$
- **47.** $-9.11 \times 10^{-2} = -0.911$
- **48.** $4 \times 10^{-6} = 0.000004$

**To compute with numbers written in scientific notation, separate the parts, compute, then recombine.**

- **Example:** $(3.14 \times 10^5)(2) = 6.28 \times 10^5$
- **Example:** $(4.28 \times 10^6)^2 = 2.20 \times 10^{12}$
- **Example:** $(2.01 \times 10^{-3}) = 8.04 \times 10^{-5}$

- **49.** $10^{-4} \times 10^{-2} = 10^{-6}$
- **50.** $10^{-10} = 10^{-10}$

- **51.** $1.86 \times 10^4 = 1.86 \times 10^4$
- **52.** $2.6 \times 10^5 = 2.6 \times 10^5$
- **53.** $1.8 \times 10^{-8} = 1.8 \times 10^{-8}$
- **54.** $(4 \times 10^{-3})^2 = 1.6 \times 10^{-6}$
- **55.** $(2.5 \times 10^{-2})^{-1} = 4 \times 10^2$
- **56.** $(-2.92 \times 10^3)(4.1 \times 10^7) = -1.208 \times 10^{11}$

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One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied by the Mathematics Diagnostic Testing Project, Gavlev Center Suite 101, UCLA, 405 Hilgard Ave., Los Angeles, CA 90024.
D. Simplification of square roots

\[ \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \text{ if } a \text{ and } b \text{ are both non-negative (} a \geq 0 \text{ and } b \geq 0\).}

Example:

- \[\sqrt{32} = \sqrt{16 \cdot 2} = 4 \sqrt{2}\]
- \[\sqrt{75} = \sqrt{25 \cdot 3} = 5 \sqrt{3}\]
- \[\sqrt{x^6} = x^3 \quad \text{if } x \geq 0, \quad \sqrt{x^6} = |x^3| \quad \text{if } x < 0\]

Note: \(\sqrt{a} = b\) means (by definition) that:
1) \(b^2 = a\), and
2) \(b \geq 0\)

57 to 69: Simplify (assume all square roots are real numbers):

57. \(\sqrt{81} = 9\)
58. \(-\sqrt{81} = -9\)
59. \(2 \sqrt{9} = 6\)
60. \(4 \sqrt{5}\)
61. \(\sqrt{16} = 4\)
62. \(3 \sqrt{12} = 6 \sqrt{3}\)
63. \(\sqrt{64} = 8\)
64. \(4 \sqrt{2}\)
65. \(\sqrt{x^2} = x\)
66. \(\sqrt{4x^6} = 2x^3\)
67. \(\sqrt{a^2} = a\)
68. \(\sqrt{36} = 6\)
69. \(\sqrt{16/3} = \frac{4}{\sqrt{3}}\)

F. Multiplying square roots

\[\sqrt{a} \cdot \sqrt{b} = \sqrt{ab} \text{ if } a \geq 0 \text{ and } b \geq 0\]

Example:

- \[\sqrt{6} \cdot \sqrt{24} = \sqrt{144} = 12\]
- \[\sqrt{2} \cdot \sqrt{6} = \sqrt{12} = 2 \sqrt{3}\]
- \[\sqrt{5} \cdot \sqrt{2} = \sqrt{100/3} = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}\]
- \[\sqrt{3} \cdot \sqrt{7} = \sqrt{21}\]

74 to 79: Simplify:

74. \(\sqrt{3} \cdot \sqrt{3} = 3\)
75. \(\sqrt{3} \cdot \sqrt{4} = 2 \sqrt{3}\)
76. \((\sqrt{3})^2 = 3\)
77. \((\sqrt{2})^2 = 2\)
78. \((\sqrt{3})^4 = 9\)

80 to 81: Find the value of \(x\):

80. \(\sqrt{4} \cdot \sqrt{9} = \sqrt{x}\)
81. \(3 \sqrt{2} \cdot \sqrt{3} = 3 \sqrt{x}\)

G. Dividing square roots

\[\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \text{ if } a \geq 0 \text{ and } b > 0\]

Example:

\[\frac{\sqrt{2}}{\sqrt{6}} = \sqrt{\frac{2}{6}} = \sqrt{\frac{1}{3}}\]

82 to 86: Simplify:

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</thead>
<tbody>
<tr>
<td>82.</td>
<td>(\sqrt{3} \div \sqrt{4})</td>
<td>(\sqrt{3})</td>
<td>(\sqrt{4})</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>83.</td>
<td>(\sqrt{9} \div \sqrt{25})</td>
<td>(\sqrt{9})</td>
<td>(\sqrt{25})</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>11</td>
<td>13</td>
<td>17</td>
<td>19</td>
<td>23</td>
<td>29</td>
<td>31</td>
</tr>
<tr>
<td>84.</td>
<td>(\sqrt{16} \div \sqrt{9})</td>
<td>(\sqrt{16})</td>
<td>(\sqrt{9})</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>14</td>
<td>16</td>
<td>19</td>
<td>22</td>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>85.</td>
<td>(\sqrt{36} \div \sqrt{4})</td>
<td>(\sqrt{36})</td>
<td>(\sqrt{4})</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>18</td>
<td>21</td>
<td>24</td>
<td>27</td>
<td>30</td>
<td>33</td>
</tr>
<tr>
<td>86.</td>
<td>(\sqrt{16} \div \sqrt{16})</td>
<td>(\sqrt{16})</td>
<td>(\sqrt{16})</td>
<td>4</td>
<td>4</td>
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If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

Example:

\[\sqrt{5} \cdot \sqrt{2} = \frac{\sqrt{10}}{\sqrt{2}} = \frac{\sqrt{10} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{20}}{2} = \frac{\sqrt{4 \cdot 5}}{2} = \frac{\sqrt{4} \cdot \sqrt{5}}{2} = \frac{2 \sqrt{5}}{2} = \sqrt{5}\]

H. Adding and subtracting square roots

Example:

\[\sqrt{5} + 2 \sqrt{5} = 3 \sqrt{5}\]
\[\sqrt{32} - \sqrt{2} = \sqrt{4 \cdot 2 - 2} = \sqrt{3} \cdot \sqrt{2}\]

0 to 73: Simplify:

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<tbody>
<tr>
<td>0.</td>
<td>(\sqrt{3} + \sqrt{3})</td>
<td>(\sqrt{3})</td>
<td>(\sqrt{3})</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>1.</td>
<td>(2 \sqrt{3} + \sqrt{27} - \sqrt{75})</td>
<td>(\sqrt{36})</td>
<td>(3 \sqrt{2})</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>2.</td>
<td>(3 \sqrt{2} + \sqrt{2})</td>
<td>(4 \sqrt{2})</td>
<td>(2 \sqrt{3})</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>3.</td>
<td>(5 \sqrt{3} - \sqrt{3})</td>
<td>(4 \sqrt{3})</td>
<td>(\sqrt{3})</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

Example:

\[\sqrt{\frac{8}{3}} = \frac{\sqrt{8}}{\sqrt{3}} = \frac{\sqrt{4 \cdot 2}}{\sqrt{3}} = \frac{2 \sqrt{2}}{\sqrt{3}} = \frac{2 \sqrt{2} \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{2 \sqrt{6}}{3}\]
A. Intersecting lines and parallel lines: If two lines intersect as shown, adjacent angles add to 180°. For example, \( a + d = 180° \). Non-adjacent angles are equal for example, \( a = c \).

If two lines, \( a \) and \( b \), are parallel and cut by a third line \( c \), forming angles \( x, y, z \) as shown, \( a + c = 180° \). \( x = y \) as \( z = x \). \( w + y = 180° \) so \( z + x = 180° \).

Example: If \( a = 3x \) and \( b = x \), find the measure of \( c \).

\[ a + b = 180°, \text{ so } 3x + x = 180°, \text{ giving } 4x = 180° \text{ or } x = 45° \]

Thus \( c = x = 45° \)

1 to 1: Given \( x = 127° \). Find the measures of the other angles:

1. \( t = \frac{x}{2} \)
2. \( y = \frac{x}{2} \)
5. \( f = 2x \)

B. Formulas for perimeter \( P \) and area \( A \) of triangles, squares, rectangles, and parallelograms

Rectangles: base \( b \), altitude \( h \)

\[ P = 2b + 2h \]

\[ A = bh \]

If a wire is bent in the shape, the perimeter is the length of the wire, and the area is the number of square units enclosed by the wire.

Example: Rectangle with \( b = 20 \) and \( h = 8 \):

\[ P = 2b + 2h = 2(20) + 2(8) = 40 + 16 = 56 \text{ units} \]

\[ A = bh = 20 \times 8 = 160 \text{ sq. units} \]

A square is a rectangle with all sides equal, so the formulas are the same (and simpler) if the side length \( s \) is:

\[ P = 4s \]

\[ A = s^2 \]

Example: Square with side 12 cm has \( P = 4s = 4 \times 12 = 48 \text{ cm} \)

\[ A = s^2 = 12^2 = 144 \text{ cm}^2 \]

A parallelogram with base \( b \) and height \( h \) has \( A = bh \).

If the other side length \( a \) is, then \( P = 2a + 2b \)

C. Formulas for area \( A \) and circumference \( C \) of a circle

A circle with radius \( r \) (and diameter \( d = 2r \) has distance around (circumference) \( C = \pi d \) or \( C = 2\pi r \)

(If a piece of wire is bent into a circular shape, the circumference is the length of wire.)

Example: A circle with radius \( r = 7 \) has \( d = 2r = 14 \) and exact circumference \( C = 2\pi \times 7 = 14\pi \text{ units} \).

If \( r \) is approximately by \( \frac{22}{7} \),

\[ C \approx \frac{22}{7} \times 14 \approx 44 \text{ units} \approx \]

If \( r \) is approximately by \( 3.14 \), the approximate \( C \approx 3.14 \times 7 \approx 22 \text{ units} \)

The area of a circle is \( A = \pi r^2 \)

Example: If \( r = 5 \),

\[ A = \pi r^2 = \pi \times 5^2 = 78.5 \text{ sq. units} \]

14 to 15: Find \( C \) and \( A \) for each circle:

14. \( r = 5 \) units

15. \( r = 10 \) feet

16. \( d = 4 \) km

D. Formulas for volume \( V \)

A rectangular solid (box) with length \( l \), width \( w \), and height \( h \), has volume \( V = lwh \).

Example: A box with dimensions \( 3 \), \( 7 \), and \( 11 \) has what volume?

\[ V = lwh = 3 \times 7 \times 11 = 231 \text{ cu. units} \]

A cube is a box with all edges equal. If the edge is \( e \),

the volume \( V = e^3 \).

Example: A cube has edge \( 4 \) cm.

\[ V = e^3 = 4^3 = 64 \text{ cm}^3 \]

A right circular cylinder with radius \( r \) and altitude \( h \) has

\[ V = \pi r^2 h \]

Example: A cylinder has \( r = 5 \)

and \( h = 14 \). The exact volume is

\[ V = \pi r^2 h = \pi \times 5^2 \times 14 = 560 \pi \text{ cu. units} \]

If \( \pi \) is approximated by \( \frac{22}{7} \),

\[ V = \frac{22}{7} \times 25 \times 14 \approx 1100 \text{ cu. units} \]

If \( \pi \) is approximated by \( 3.14 \),

\[ V = 3.14 \times 25 \times 14 \approx 1130.4 \text{ cu. units} \]

A sphere (ball) with radius \( r \) has volume \( V = \frac{4}{3} \pi r^3 \)

Example: The exact volume of a sphere with radius \( 6 \) in.

\[ V = \frac{4}{3} \pi \times 6^3 = \frac{4}{3} \pi \times 216 = 288 \pi \text{ in}^3 \]

17 to 24: Find the exact volume of each of the following solids:

17. Box, \( 6 \times 8 \times 9 \)

18. Box, \( 12 \times 5 \times 8 \)

19. Cube with edge \( 10 \)

20. Cube, edge \( 5 \)

21. Cylinder with \( r = 5 \), \( h = 10 \)

22. Cylinder, \( r = \sqrt{3} \), \( h = 2 \)

23. Sphere with radius \( r = 2 \)

24. Sphere with radius \( r = 3 \)

E. Sum of the interior angles of a triangle: the three angles of any triangle add to 180°

Example: Find the measures of angles \( G \) and \( A \)

\( \angle G \) (angle \( G \) is marked) to show its measure is \( 90° \).

\( \angle A \) is \( 180° - 90° = 90° \)

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One of a series of worksheets designed to provide remedial practice. Coordinated with topics on diagnostic tests supplied by the Mathematics Diagnostic Testing Project, Gayley Center Suite 304, UCLA, 405 Hilgard Ave., Los Angeles, CA 90024.
25 to 29: Given two angles of a triangle, find the measure of the third angle.

25. 30°, 60°
26. 120°, 36°
28. 42°, 83°
27. 90°, 17°
29. 68°, 44°

II. Isosceles triangles

An isosceles triangle is defined to have at least two sides with equal measure. The equal sides may be marked: 

or the measures may be given:

30 to 35: Is the triangle isosceles?

30. Sides 4, 4, 5
31. Sides 7, 6, 7
32. Sides 8, 8, 8
33. Sides 3, 4, 5
34. Sides 9, 9, 10
35. Sides 2, 2, 1

The angles which are opposite the equal sides also have equal measures (and all three angles add to 180°).

example: Find the measures of \( \angle A \) and \( \angle C \), given \( \angle B = 80° \); \( \angle A + \angle B + \angle C = 180° \), and \( \angle A = \angle C \).

36. Find measures of \( \angle A \) and \( \angle B \), if \( \angle C = 30° \).
37. Find measures of \( \angle B \) and \( \angle C \), if \( \angle A = 30° \).
38. Find measure of \( \angle A \).

39. If the angles of a triangle are \( 30°, 60° \), and \( 90° \), can it be isosceles?
40. If two angles of a triangle are \( 55° \) and \( 60° \), can it be isosceles?

If a triangle has equal angles, the sides opposite these angles also have equal measures.

example: Find the measures of \( \angle A \), \( \angle B \), and \( \angle C \), given this figure, and \( \angle C = 40° \); \( \angle A = \angle B \).

41. Can a triangle be isosceles and have a 90° angle?
42. Given \( \angle A = \angle B = 60° \) and \( \angle C = 60° \), find the measure of \( \angle F \). Is \( \angle F \) a right angle?

50. Find \( x \) and \( y \):

51 to 56: Each line of the chart lists two sides of a right triangle. Find the length of the third side:

51. 6, 8
52. 8, 10
53. 5, 12
54. \( \sqrt{2} \), \( \sqrt{3} \)
55. 5, 12
56. 15, 5

If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

example: The ratio \( \frac{a}{b} \), or \( \frac{x}{y} \), is the same as \( \frac{a}{y} \), or \( \frac{x}{y} \). Thus, \( \frac{a}{x} \) = \( \frac{y}{x} \), and \( \frac{b}{y} \), or \( \frac{y}{x} \). Each of these equations is called a proportion.

57 to 59: Is a triangle right, if it has sides:

57. 4, 5, 6
58. 6, 8, 10
59. 60, 61, 11

Answers:

1. 120°
2. 50°
3. 50°
4. 120°
5. 36°
6. 30°, 60°, 90°
7. 30°, 60°, 90°
8. 30°, 60°, 90°
9. 120°, 60°, 60°
10. 120°, 60°, 60°
11. 150°, 60°, 60°
12. 120°, 60°, 60°
13. 150°, 60°, 60°
14. 150°, 60°, 60°
15. 120°, 60°, 60°
16. 100°
17. 100°
18. 100°
19. 100°
20. 100°
21. 100°
22. 60°
23. 30°
24. 90°
25. 90°
26. 90°
27. 90°
28. 90°
29. 90°
30. 90°
31. yes
32. yes
33. yes
34. yes
35. can't be
36. 75° each
37. 120°, 30°
38. 60°
39. no
40. no
41. yes
42. 60°, 60°
43. \( \angle A \), \( \angle B \), \( \angle C \), \( \angle D \), \( \angle E \), \( \angle F \)
44. \( \frac{a}{b} \) = \( \frac{c}{d} \)
45. \( x = \sqrt{17} \), \( y = 4 \)
46. 10, 10
47. 9
48. 8
49. 6
50. 5
51. 3
52. 15
53. 5
54. 10
55. 12
56. 5

Example: Is a triangle right, if it has sides 20, 29, 21:

\( 20^2 + 21^2 = 22^2 \), so it is a right triangle.

Example: In a triangle right, if it has sides:

57. 4, 5, 6
58. 5, 12, 13
59. 6, 8, 10
A. Arithmetic, percent, and average

1. What is the number, which when multiplied by 32, gives 32 \cdot 46?  
2. If you square a certain number, you get 9^2. What is the number?  
3. What is the power of 36 that gives 36^2?  
4. Find 3\% of 36.  
5. 55 is what percent of 88?  
6. What percent of 55 is 88?  
7. 45 is 80\% of what number?  
8. What is 8.3\% of $7000?  
9. If you get 36 on a 40-question test, what percent is this?  
10. The 3200 people who vote in an election are 40\% of the people registered to vote. How many are registered?  

11 to 13: Your wage is increased by 20\%, then the new amount is cut by 20\% (of the new amount).  
11. Will this result in a wage which is higher than, lower than, or the same as the original wage?  
12. What percent of the original wage is this final wage?  
13. If the above steps were reversed (20\% cut followed by 20\% increase), the final wage would be what percent of the original wage?  

14 to 16: If A is increased by 25\%, it equals B.  
14. Which is larger, B or the original A?  
15. B is what percent of A?  
16. A is what percent of B?  

17. What is the average of 67, 36, 48, 59, and 95?  
18. If two test scores are 85 and 60, what minimum score on the next test would be needed for an overall average of 80?  
19. The average height of 49 people is 68 inches. What is the new average height if a 78-inch person joins the group?  

B. Algebraic substitution and evaluation

20 to 24: A certain TV uses 75 watts of power, and operates on 120 volts.  
20. Find how many amps of current it uses, from the relationship: volts times amps equals watts.  
21. 1000 watts = 1 kilowatt (kw). How many kilowatts does the TV use?  
22. Kw times hours = kilowatt-hours (kwh). If the TV is on for six hours a day, how many kw of electricity are used?  
23. If the set is on for six hours every day of a 30-day month, how many kw are used for the month?  
24. If the electric company charges 6\% per kwh, what amount of the month's bill is for TV power?  
25 to 33: A plane has a certain speed in still air, where it goes 1350 miles in three hours.  
25. What is its (still air) speed?  
26. How far does the plane go in 5 hours?  
27. How far does it go in x hours?  
28. How long does it take to fly 2000 mi.?  
29. How long does it take to fly y mi.?  
30. If the plane flies against a 50 mph headwind, what is its ground speed?  
31. If the plane flies against a headwind of z mph, what is its ground speed?  
32. If it has fuel for 7.5 hours of flying time, how far can it go against the headwind of 50 mph?  
33. If the plane has fuel for t hours of flying time, how far can it go against the headwind of z mph?  

C. Ratio and proportion

34 to 35: x is to y as 3 is to 5.  
34. Find y when x is 7.  
35. Find x when y is 7.  

36 to 37: A is proportional to F, and F = 56 when a = 1\% .  
36. Find a when F = 1\% .  
37. Find F when a = 1\% .  

38 to 39: Given 3x = 4y .  
38. Write the ratio x:y as the ratio of two integers.  
39. If x = 3, find y.  

40 to 41: x and y are numbers, and two x's equal three y's.  
40. Which of x or y must be larger?  
41. What is the ratio of x to y?  

42 to 44: Half of x is the same as one-third of y.  
42. Which of x and y is the larger?  
43. Write the ratio x:y as the ratio of two integers.  
44. How many x's equal 30 y's ?  

D. Problems leading to one linear equation

45. 36 is three-fourths of what number?  
46. What number is 3/4 of 36?  
47. What fraction of 36 is 15?
48. 2/3 of 1/6 of 3/4 of a number is 12. What is the number?
49. Half the square of a number is 18. What is the number?
50. 81 is the square of twice what number?
51. Given a positive number x. Two times a positive number y is at least four times x. How small can y be?
52. Twice the square root of half a number is 2x. What is the number?
53 to 55: A gathering has twice as many women as men. W is the number of women and M is the number of men.
53. Which is correct: 2W = W or M = 2W?
54. If there are 12 women, how many men are there?
55. If the total number of men and women present is 54, how many of each are there?
56. $12,000 is divided into equal shares. Bob gets four shares, Bill gets three shares, and Ben gets the one remaining share. What is the value of one share?

E. Problems leading to two linear equations
57. Two science fiction coins have values x and y. Three x's and five y's have a value of 75¢, and one x and two y's have a value of 27¢. What is the value of each?
58. In mixing x gm of 3% and y gm of 8% solutions to get 10 gm of 5% solution, these equations are used:
   \[0.03x + 0.08y = 0.05(10),\] and
   \[x + y = 10\]
   How many gm of 3% solution are needed?

F. Geometry
59. Point X is on each of two given intersecting lines. How many such points X are there?
60. On the number line, points P and Q are two units apart. Q has coordinate x. What are the possible coordinates of P?

61 to 62:
61. If the length of chord AB is x and the length of CB is 16. What is AC?
62. If AC = y and CB = z, how long is AB (in terms of y and z)?

63 to 64:
63. The base of a rectangle is three times the height.
64. Find the height if the base is 20.
64. Find the perimeter and area.

65. In order to construct a square with an area which is 100 times the area of a given square, how long a side should be used?
66 to 67: The length of a rectangle is increased by 25% and its width is decreased by 10%.
66. Its new area is what percent of its old area?
67. By what percent has the old area increased or decreased?
68. The length of a rectangle is twice the width. If both dimensions are increased by 2 cm, the resulting rectangle has 84 cm² more area. What was the original width?
69. After a rectangular piece of knitted fabric shrinks in length one cm and stretches in width 2 cm, it is a square. If the original area was 40 cm², what is the square area?

70. This square is cut into two smaller squares and two non-square rectangles as shown. Before being cut, the large square had area \((a+b)^2\). The two smaller squares have areas a² and b². Find the total area of the two non-square rectangles. Show that the areas of the 4 parts add up to the area of the original square.